

# Project 5 21.11.09

## Question 1:

$$\sigma(u) = \frac{e^u}{1+e^u} \quad \text{sigmoid function.}$$

$$L(\beta) = \frac{1}{N} \sum_{i=1}^N l(y_i, \sigma(\hat{x}_i^T \beta))$$

$$h_i(\hat{x}_i^T \beta) = h_i(u) = l(y_i, \sigma(u))$$

$$L'(\beta) = \frac{1}{N} \sum_{i=1}^N L'_i(\beta)$$

★ remember:

$$l(p, q) = -p \log(q) - (1-p) \log(1-q)$$

$$L_i(\beta) = h_i(\hat{x}_i^T \beta)$$

$$L'_i(\beta) = h'_i(\hat{x}_i^T \beta) \cdot \hat{x}_i$$

$h'_i(u) \rightarrow \text{solve}$

$$\begin{aligned} h_i(u) &= l(y_i, \sigma(u)) = -y_i \log\left(\frac{e^u}{1+e^u}\right) - (1-y_i) \log\left(1 - \frac{e^u}{1+e^u}\right) \\ &= -y_i \left[ \log(e^u) - \log(1+e^u) \right] - (1-y_i) \left[ \log(1) - \log\left(\frac{1}{1+e^u}\right) \right] \\ &= -y_i u + y_i \log(1+e^u) + \log(1+e^u) - y_i \log(1+e^u) \\ h_i(u) &= \log(1+e^u) - y_i u \end{aligned}$$

$$h'_i(u) = \frac{e^u}{1+e^u} - y_i$$

$$L'_i(\beta) = \left[ \frac{e^{\hat{x}_i^T \beta}}{1+e^{\hat{x}_i^T \beta}} - y_i \right] \hat{x}_i \quad (1/N)$$

$$L'(\beta) = \frac{1}{N} \sum_{i=1}^N h'_i(\hat{x}_i^T \beta) \hat{x}_i$$

$$= \frac{1}{N} \sum_{i=1}^N (\sigma(\hat{x}_i^T \beta) - y_i) \cdot \hat{x}_i$$